Rebalancing the Simon Fraser University's Academic Pension Plan's Balanced Fund: A Case Study

by

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Abstract

The purpose of the paper is to investigate the rebalancing strategy for Simon Fraser University's Academic Pension Plan's Balanced Fund. First, we examine performances of a "no rebalancing" fund and rebalanced funds with different rebalancing frequencies and thresholds based on the historic data. The results show that the rebalancing frequency and thresholds do not significantly affect the performance of the portfolio. Additionally, the rebalanced portfolios significantly outperform the "no rebalancing" portfolio. More important, we examine whether the conclusion from one historic simulation holds in 10,000 Monte Carlo simulations based on historic means, variances and co-variances and two sets of hypothetical means. The results indicate that the higher rebalancing frequency and smaller threshold will reduce cumulative wealth of rebalanced portfolios and reduce risk.

Key Words:

Rebalancing, Asset Allocation

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Introduction

Rebalancing is the process of resetting the weights of assets in one's portfolio by periodically selling or buying each asset class in order to maintain its target asset allocation. It is a portfolio-monitoring strategy in place to handle the ever-changing financial market. Tokat and Wicas (2007) state that the biggest benefit of rebalancing is to maintain the investors' original investment policy. If a portfolio is never rebalanced, it will drift from its target asset allocation to higher return and higher risk assets, acquiring return and risk characteristics that may be inconsistent with an investor's goals and preferences. O'Brien (2006) argues that the process of rebalancing is actually

counterintuitive if the principle of rebalancing is selling high return high risk assets and buying low return low risk assets. Usually, the investors believe the asset that is going up will continue to go up and vice versa. Rebalancing can also reduce the loss of the portfolio when investors' intuitions are opposite to real market direction.

However, it is unclear whether rebalancing can improve the return of the portfolio. Tokat and Wicas (2007) point out that the answer depends on the market environment. They classify market environment into three types: trending markets, mean-reverting markets and random-walk markets. In the trending markets, if equity returns beat bond returns every period, rebalancing implies continually selling the strong performer and investing in the weaker performer. Rebalancing will produce a lower return than a portfolio that was never rebalanced, and the more frequent rebalancing will produce a lower return. In the mean-reverting markets, price increases are followed by price declines, and vice versa. The return of rebalanced portfolio is enhanced because we buy an asset after its price has decreased and sell asset after it has increased. The random walk is a more realistic market pattern since the returns of asset are unpredictable. In the random market, the more frequent rebalanced portfolios have lower return and lower risk.

The rebalancing strategy hinges on two main factors: threshold and frequency. Threshold measures the deviation from target allocation to the current allocation. Frequency refers how often the rebalancing needs to be considered. For any rebalancing events, two conditions need to be satisfied. First, is it a rebalancing time? Second, did the current allocation trigger the threshold? For an instance of a rebalancing strategy with quarterly frequency and 3% threshold, first, we need to check whether it is at January, April, July

or October. If it is, we get into the second step to check whether any asset weight drifts 3% more or less than the target weight.

The purpose of the paper is to investigate the rebalancing strategy for Simon Fraser University's Academic Pension Plan's Balanced Fund. First, we look at the performances of a "no rebalancing" fund and rebalanced funds with monthly, quarterly, semi-annual and annual rebalancing frequency. Second, we loosen and tighten the bounds of the rebalanced portfolio to examine the threshold effect. This portion of the study is based on the historic data. More important, we examine whether the conclusion from one historic simulation holds in 10,000 Monte Carlo simulations based on historic means, variances and co-variances. Finally, we conduct Monte Carlo simulation based on two sets of hypothetical means. One set of hypothetical means is reasonable and another set of hypothetical means value is more extreme where the return of US equities is 20% and the return of Canadian Fixed Income is 5%.

The paper proceeds as follows. Section 2 outlines methodology of the study. Section 3 shows the result of rebalancing simulations. Section 4 contains summaries.

1. Methodology

In this section, we present the methodology used to investigate rebalancing strategies for Simon Fraser University's (SFU) Academic Pension Plan's Balanced Fund. The Balanced Fund is not charged transactions costs when it is rebalanced. Hence, we do not consider transactions costs in this paper. The results may well be different if we did.

The rebalancing strategies involve two factors: how often the portfolio needs to be rebalanced and how wide the rebalancing bounds are. First, given the normal allocation by the SFU Balanced Fund guideline, we conduct a simulation of a "no rebalancing" portfolio one time through the time horizon from January 1970 to June 2012. Second, we conduct rebalancing simulations with the guideline bounds using monthly, quarterly, semi-annual and annual rebalancing strategies to compare the performance of the portfolio with different rebalancing frequencies. Third, we loosen and tighten the bounds with the aforementioned five frequencies from January 1970 to June 2012.

More important, we examine whether the conclusion from one historic simulation holds in a full scale Monte Carlo simulation where the exercise to repeated 10,000 times, hoping that the conclusion can be applied to the real portfolio management world and are not defined to a particular time period. We assume the expected mean, variance and covariance for each asset class's returns are the same with those in history; Asset class returns follow a random walk and are joint normal distributed in next 42.5 years. Based on the above assumption, we simulate 510 monthly returns and conduct rebalancing simulations with them. This process is repeated 10,000 times.

However, we note that the historic annual compound returns of each asset are very close to each other, and it may not be a reasonable case for a new 42.5 years. We adjust the annual compound returns of Canadian Equities, US Equities, EAFE Equities, Global Equities and Canadian Fixed Income from 9.21%, 9.75%, 9.37%, 9.12% and 9.27% to 9.5%, 10%, 8%, 9% and 7%, respectively. This gives a greater spread between the equities mean and fixed income mean. The other assumptions remain unchanged. The

simulations with different bounds and rebalancing frequencies are conducted 10,000 times again. Finally, to examine an extreme case of different asset returns, we run the simulations 10,000 times where annual compound returns of US Equities and Canadian Fixed Income are 20% and 5%, respectively.

The rebalancing rule is as follows. Suppose we have one dollar in our portfolio, and the initial weights or values of each asset satisfy the target weights. The original data are monthly rate of returns of each asset. The value of each asset in the next period can be calculated directly by multiplying the current value by the one plus the rate of returns. After that, we can get the total value of the portfolio and weights of each asset in the portfolio. Next, we check whether the portfolio weights trigger the rebalancing conditions of time and bounds. If it does, we rebalance our portfolio back to the target weights. If it doesn't, we continue to get the value of each asset in the next month. This process is repeated until the end of time horizon. In the "no rebalancing" simulation, we skip the rebalancing check step and follow the rest of steps. Note that we assume no transaction cost in our simulations.

The following is a numerical example for monthly rebalancing simulation. Suppose we have one dollar in portfolio initially. The target allocation is 50% in stock and 50 % in bonds, so we have 0.5 dollars in stock and 0.5 dollars in bonds. The stock return in next month is 20% and the bond return is 8%. Then we get 0.6 dollar in stock and 0.54 dollars in bonds. Suppose this triggers the rebalancing condition and the portfolio needs to be rebalanced to its target allocation. After the rebalancing, it ends up with 0.57 dollars in stock and 0.57 dollars in bond since the total value of the portfolio is 1.14 dollars.

2. Data and Descriptive Statistics

In this section, we introduce the Simon Fraser University Academic Pension Plan's Balanced Fund guideline and the data used in our rebalancing simulation.

Table 1 exhibits the guideline of the SFU Balanced Fund. The fund is composed of five asset classes: Canadian Equities, US Equities, EAFE Equities, Global Equities and Fixed Income. The normal allocations are target weights for assets. When the weight of any asset triggers the upper or lower bounds of range, the whole portfolio will be brought back to the normal allocation. In the portfolio, all maximum or minimum allowable weights are 3% higher or lower than the normal allocation weight. The deviations from maximum or minimum allowable weight to normal allocation weight are called the threshold. In this case, the threshold is 3%.

Table 1 here

In this project, the representative indexes of each asset class are: S&P TSX Equity TR, S&P 500(WSJ) CAD, MSCI EAFE GR CAD, MSCI WORLD GR CAD, DEX Universe, accordingly. All the indices data are obtained from MorningStar Encorr. The selected data cover a period from January 1970 to June 2012. Since the data of DEX Universe starts from December 1979, DEX Long Term Bond index is chosen as an alternative to represent the missing part of DEX Universe. Returns of each asset class are all expressed in Canadian dollars and both dividends and capital gain/loss have been taken into consideration.

The selected data are monthly arithmetic returns. Table 2 shows the historic asset returns and the correlations between the asset classes. We can see that equities have higher returns and higher volatilities than those of Canadian Fixed Income. Among the four equities, US equity has the second lowest volatility but the highest annual compound return, which results in the greatest cumulative wealth; while Global Equity has the lowest return with lowest volatility. In term of minimum return, Canadian Equity has the worst minimum monthly return and EAFE equities have the best maximum monthly return. The return range of the fixed income is much narrower than that of the equities. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period. In our case, we invest one dollar in January 1970, the cumulative wealth is the value the investment in June 2012. Annual compound return is calculated to give a more direct sense on returns on a yearly basis. Surprisingly, there is no distinguishable difference between equities and fixed income in term of annual compound returns. As indicted in the correlation table, the correlations between equities are higher than those between equities and fixed income. The Global Equity is highly correlated with other equities.

Table 2 here

3. Results

In this section, we present the result of one historic simulation and the 10,000 Monte Carlo simulations. To assess the performance of the portfolio with different rebalancing strategies, we compare volatility, the minimum return, cumulative wealth and the number of rebalances that occur. In the 10,000 Monte Carlo simulations, we compare the average

of those statistics. Additionally, the minimum, 5% quantile, median, 95% quantile, maximum and standard deviation of cumulative wealth are also considered in the Monte Carlo simulations. Ideally, a rebalancing strategy should provide a high mean return, a high cumulative wealth, a low absolute value of minimum return, low volatility and low number of rebalancing occurrences.

4.1 Historic Performance of a "No Rebalancing" Portfolio

Table 3 shows the portfolio statistics in a "no rebalancing" case, the mean return is 0.79%, volatility of return is 2.91%, the minimum return is -12.76%. The cumulative wealth of a "no rebalancing" portfolio is $44.38.

Table 3 here

We examine how far one asset may drift from its target asset allocation if we never rebalance our portfolio. Table 4 shows the maximum and minimum weights each asset could reach. From the table, we can see that Canadian fixed income has the greatest downside deviation while EAFE equities have the greatest upside deviation. Table 5 describes the mix of the portfolio weights when one asset class moves to its extreme. For example, Canadian Equities reach its minimum weight of 14.27%, the weights of US equities, EAFE equities; Global equities and Canadian Fixed Income are 25.33%, 22.89%, 13.29% and 24.22%, respectively.

Tables 4 and 5 here

4.2 Historic Performance of Rebalanced Portfolios

Table 6 illustrates the results of the historic rebalancing simulations over the 42.5 year period using rebalancing monthly, quarterly, semi-annual and annual rebalancing frequencies with a 3% threshold (i.e., rebalancing whenever asset weights drift more or less than 3% from original asset allocation). Portfolio mean returns are 0.82% which is higher than that of the "no rebalancing" portfolio. The volatilities of return are 2.74% which is lower than that in "no rebalancing" case. (The return and volatility are rounded to two decimal places). The minimum returns of rebalanced portfolio are very close but all are lower than the minimum return in a "no rebalancing" portfolio. Under different rebalancing frequencies, we find that the higher frequency we pick, the more rebalancing events occur. However, the rebalancing frequency does not affect the mean returns, volatility of returns, cumulative wealth and volatility of the portfolio.

Table 6 here

4.3 Sensitivity Test: Changing the Threshold

For the SFU Balanced Fund, the 3% range is chosen as the rebalancing threshold. We change the tolerance range from 3% to 1%, 2%, 4% and 5%.

Tables from 7, 8, 9 and 10 exhibit the portfolio rebalancing simulation result with threshold 1%, 2%, 4% and 5%, respectively. The sensitivity test does not show any significant difference in mean returns, volatility.

The number of times the portfolio was rebalanced decreases when we decrease rebalancing frequency or enlarge the threshold. The differences of the rebalancing instance number under different rebalancing frequency shrink when we increase the threshold because the threshold plays a more important role in making rebalancing decision. Another interesting result is that the absolute values of minimum returns become greater when we increase the threshold our portfolio. This indicates that narrowing the bounds is an effective way to reduce the minimum return.

Tables 7, 8, 9 and 10 here

Table 11 summaries the cumulative wealth of the portfolio under different scenario. From this table, there is no pattern to show that the cumulative wealth is affected by the rebalancing frequency. Meanwhile, there is weak evidence that the larger threshold results in higher cumulative wealth.

Table 11 here

4.4 Comparison between Rebalanced Portfolios and a "No Rebalancing" Portfolio

The cumulative wealth of the rebalanced portfolio ranged from $52.43 to $56.52, which all are significantly higher than that of a "no rebalancing" portfolio. We observe the detailed information of simulation evolutions in the rebalanced portfolios and "no rebalancing" portfolio. Table 12 shows the cumulative wealth of annual rebalancing portfolios with 3% threshold and the "no rebalancing" portfolio from 1970 to 2012.

Before 2005, the values of the two portfolios are very close. However, at 2005, the rebalanced portfolio has significantly higher value than a "no rebalancing" portfolio.

Table 12 here

What happened from 2000 to 2005? The table 13 shows the detailed information of the two portfolios. The rebalancing events happen on January 2000, January 2002, January 2003 and January 2004. In 2000, the rebalanced portfolio allocates more money in fixed income and Canadian equities than "no rebalancing" portfolio did. At the same time, the rebalanced portfolio put much less money in US equities, EAFE equities and global equities. In next 5 years, US equity, EAFE and global equity market crashed. The rebalanced portfolio successfully avoided this disaster due to its rebalancing strategy that underweighting those three sectors. Moreover, looking at the values of each asset in two portfolios at January 2000, we can clearly see that the US equities, EAFE equities, global equities have better returns in the past 30 years than others since they far deviate above the target weights.

Table 13 here

Tokat and Wicas (2007) pointed out that in a mean-reverting market, a portfolio's buy-and-sell decisions are generally well timed when rebalancing it. The return of rebalanced portfolio is higher than that of a "no rebalancing" portfolio. Due to the financial crisis, the markets show the characteristics of mean reverting market in last 42.5 years time horizon. That's why the rebalanced portfolio can outperform the no rebalanced portfolio.

What's more, one drawback of rebalancing shows up in this scenario. The return of fixed income in rebalanced portfolio increases by only 14%, at the same time, the one in the "no rebalancing" portfolio increased 50%. Ironically, the "no rebalancing" portfolio did not put a lot of money in fixed income in 2000, so the gain in fixed income cannot cover the loss in US equities, EAFE equities and global equities. Besides, the Canadian equities in both portfolios gained 18% in five years. The rebalanced portfolio made more money than other did since it allocates more money in Canadian equities in 2000.

4.5 Monte Carlo Simulations Based on the Historic Means

In the first Monte Carlo rebalancing simulations, we assume the expected means, variances and co-variances for each asset return are the same with those in history, all assets returns follow random walk and are joint normal distributed. Tables 14, 15 and 16 show the result of Monte Carlo rebalancing simulations with different frequencies and thresholds.

Comparing with the results of rebalanced portfolios in one-time historic simulation, the mean returns remain the same and the volatilities of returns increase in Monte Carlo simulation. However, the absolute value of minimum returns become lower because the distribution of historic returns are not normal and they have a fatter tail than normal distribution.

In term of the rebalancing effect, the threshold and the rebalancing frequency still do not affect mean return and volatility of return (They might do affect if they are rounded to more decimal places). However, we can barely see that the absolute value of minimum

return and cumulative wealth increase when we loosen the bounds or lengthen the frequency. Also, it is amazing to see that the cumulative wealth can change so much in different simulations. The 5% quantile, median, 95% quantile and standard deviation of the cumulative wealth show the exactly same pattern as the cumulative wealth does. The number of rebalancing instances increases when we tighten the rebalancing conditions.

The absolute value of minimum return and volatility of return in "no rebalancing" portfolio are higher than those in rebalanced portfolios, which is the same with that in one-time historic simulation. Moreover, the volatility of cumulative wealth in "no rebalancing portfolio" is higher than that in the rebalanced portfolios. Those results indicate that rebalancing can effectively reduce the risk of the portfolio. However, the relationship of returns between rebalanced and "no rebalancing" portfolio do not hold anymore.

In the Monte Carlo simulation, the mean return of "no rebalancing" portfolio increases to 0.82 which is equal to those in rebalanced portfolios. The cumulative wealth of the "no rebalancing" portfolio is slightly higher than the rebalancing portfolio.

Tables 14, 15, 16 here

The reason is that the Monte Carlo simulation assumes market is random walk but the real world market may not. We implement the serial correlation test on historic data. The results show that the Canadian equities, EAFE equities, Global equities and Canadian Fixed Income returns have significant series correlation. Due to the market environment difference between real world and simulation, the rebalanced portfolios have lower

cumulative wealth than the "no rebalancing" portfolio in the Monte Carlo Simulation, which contradicts the results in historic simulation.

4.6 Monte Carlo Simulation Based on Hypothetical Mean Returns

We note that the historic annual compound returns of each asset are very close to each other, and it may not be a reasonable case for a new 42.5 years. We increase the annual compound returns of Canadian equities and US equities from 9.21% and 9.75% to 9.5% and 10%; and decrease the annual compound returns of EAFE equities, global equities and Canadian fixed income from 9.37%, 9.12% and 9.27% to 8%, 9% and 7% respectively and maintain the other assumptions. This gives a greater spread between the equities mean and fixed income mean. Tables 17, 18 and 19 exhibit the result of rebalancing simulation with different threshold and rebalancing frequencies based on reasonable means.

Comparing with the simulations based on the historic means, the mean returns and cumulative wealth went down because we slightly shift up the returns of Canadian equities and US equities but dramatically shift down the return of EAFE equities, global equities and Canadian fixed income.

In term of rebalancing effect, we can see that loosening the bounds or lengthening the frequency increase the cumulative wealth but also increase the absolutely value of minimum returns and volatility of cumulative wealth. Moreover, the change of the volatilities of returns becomes observable and they slightly increase when we loosen the rebalancing conditions. The "no rebalancing" portfolio still has the highest mean returns,

cumulative wealth, volatilities of returns, absolute value of minimum return and volatility of cumulative wealth since " no rebalancing" portfolio is a case with extreme loose rebalancing constrains.

Tables 17, 18 and 19 here

After we increase the difference between equities return and fixed income return, the change of observations becomes more significant and the pattern of observations trends to be more obvious. Consequently, we conduct a Monte Carlo simulation at an extreme case where the annual compound returns of US equities and Canadian fixed income are 20% and 5% respectively. Tables 20, 21 and 22 show the result of simulation with the extreme value hypothetical means. The more frequent rebalancing reduces the return but reduces the risk. As a result, the mean return, cumulative return, volatility of return, absolute value of minimum return and volatility of cumulative wealth increase when we loosen the bounds or lengthen the frequency. The "no rebalancing" portfolio has the highest corresponding five observations. Also, we find that the cumulative wealth of no rebalancing portfolio is 581.34, which is far away higher the cumulative wealth in the rebalanced portfolio. The US equities are gradually taking over the whole portfolio.

Tables 20, 21 and 22 here

5. Summary

In this paper, we investigate the rebalancing strategy for Simon Fraser University's Academic Pension Plan's Balanced Fund. The Balanced Fund is not charged transactions

costs when it is rebalanced. Hence, we do not consider transactions costs in this paper. The results may well be different if we did.

In the one-time historic simulation, we find that the rebalancing frequency and threshold do not affect mean return, accumulative wealth and volatility of the portfolio. There is weak evidence showing that the minimum return of portfolio increases in absolute value by either loosening the bounds or lengthening the rebalancing frequency. Comparing with the "no rebalancing" portfolio, all the rebalanced portfolios have lower volatility and absolute value of minimum return but higher mean returns and cumulative wealth, which indicates the rebalanced portfolio outperform the "no rebalancing" portfolio in all respects.

In the Monte Carlo simulations, we find that the more frequent rebalancing reduces the return and reduces the risks. As a result, the mean return, cumulative return, volatility of return, absolute value of minimum return and volatility of cumulative wealth increase when we loosen the bounds or lengthen the frequency. The "no rebalancing" portfolio has the highest corresponding five observations. The results are not significant in Monte Carlo simulation with historic means. However, when we increase the spread between equities return and fixed income returns, the cumulative wealth and risk reduce more when we rebalance portfolio more frequently.

References

Tokat Yesim, and Wicas Nelson. "Portfolio Rebalancing in Theory and Practice." *The Journal of Investing,* (Summer 2007, Vol.16, No2), pp.52-59.

O’Brien John. ”Rebalancing: A Tool for Managing Portfolio Risk.” J*ournal of Financial Service Professionals*, (May 2006, Vol.60, Issue3), pp.62-68.

Table 1 SFU Balanced Fund

Component Asset Classes	Percentage of Fund at Market Values		
	Normal Allocation	Range	
		Minimum	Maximum
Canadian Equities	25%	22%	28%
US Equities	15%		
EAFE Equities	15%		
Global Equities	10%		
Total Foreign Equities	40%	37%	43%
Total Equities	65%	62%	68%
Fixed Income	35%	32%	38%

Reprinted from Statement of Policies and Procedures.

Table 2 Descriptive Statistics (1970.1-2012.6)

	Canadian Equities	US Equities	EAFE Equities	Global Equities	Fixed Income
Mean return	0.85	0.87	0.85	0.81	0.76
Volatility of return	4.73	4.21	4.60	3.95	1.85
Minimum Return	-22.52	-21.11	-14.15	-16.49	-6.66
Maximum Return	16.54	16.43	18.82	15.64	9.32
Annual Compound Return	9.21	9.75	9.37	9.12	9.27
Cumulative Wealth	42.22	52.21	44.96	40.84	43.32
Correlation Matrix					
	Canadian Equities	US Equities	EAFE Equities	Global Equities	Fixed Income
Canadian Equities	1.00	0.66	0.52	0.68	0.25
US Equities	0.66	1.00	0.56	0.86	0.22
EAFE Equities	0.52	0.56	1.00	0.89	0.15
Global Equities	0.68	0.86	0.89	1.00	0.21
Fixed Income	0.25	0.22	0.15	0.21	1.00

US Equities are represented by S&P 500 TR (WSJ) CAD (1970-2012), Canadian Equities are represented by S&P TSX Equities TR (1970-2012), EAFE Equities are represented by MSCI EAFE GR CAD(1970-2012), GLOBAL Equities are represented by MSCI WORLD GR CAD(1970-2012), Fixed income are represented by DEX LT(1970-1978) and DEX Universe(1979-2012). The source of data is MorningStar Encorr.
The mean return, volatility of return, maximum return, and minimum return are stated in percent per month.
Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.

Table 3 Historic Performance of No Rebalancing Portfolio (1970-2012)

	No Rebalancing
Mean Return	0.79
Volatility of Return	2.91
Minimum Return	-12.76
Maximum Return	10.93
Annual Compound Return	9.33
Cumulative Wealth	44.38
N	0

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 4 Historic Performance of No Rebalancing Portfolio (1970-2012)

%	Canadian Equities	US Equities	EAFE Equities	Global Equities	Fixed Income
Target Weight	25.00	15.00	15.00	10.00	35.00
Minimum Weight	14.27	10.75	12.25	7.69	18.74
Maximum Weight	33.02	28.35	32.74	14.16	44.99

Minimum/Maximum weight refers that the minimum/maximum weight of every asset through the time horizon.

Table 5 Asset weights of no rebalancing portfolio when assets reach maximum/minimum weight (1970-2012)

%		Canadian Equities	US Equities	EAFE Equities	Global Equities	Fixed Income
Minimum	Canadian Equities	**14.27**	25.33	22.89	13.29	24.22
	US Equities	23.00	**10.75**	13.76	7.69	44.81
	EAFE Equities	23.38	14.34	**12.25**	8.99	41.05
	Global Equities	23.00	10.75	13.76	**7.69**	44.81
	Fixed Income	17.94	26.81	22.69	13.83	**18.74**
Maximum	Canadian Equities	**33.02**	12.15	17.52	8.91	28.40
	US Equities	15.07	**28.35**	21.70	13.86	21.02
	EAFE Equities	18.55	12.02	**32.74**	12.19	24.50
	Global Equities	16.47	27.05	23.51	**14.16**	18.81
	Fixed Income	22.30	11.31	13.54	7.86	**44.99**

Table 6 Historic Performance of Rebalancing Portfolio with 3% threshold (1970-2012)

	Monthly	Quarterly	Semi-Annual	Annual
Mean Return	0.82	0.82	0.83	0.82
Volatility of Return	2.74	2.74	2.73	2.74
Minimum Return	-10.34	-10.78	-10.34	-11.49
Maximum Return	11.70	11.96	11.35	11.35
Annual Compound Return	9.81	9.85	9.88	9.86
Cumulative Wealth	53.46	54.11	54.76	54.45
N	46	42	33	24

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 7 Historic Performance of Rebalancing Portfolio with 1% threshold (1970-2012)

	Monthly	Quarterly	Semi-Annual	Annual
Mean Return	0.82	0.82	0.82	0.82
Volatility of Return	2.74	2.73	2.72	2.74
Minimum Return	-10.20	-10.34	-10.34	-11.49
Maximum Return	12.01	11.96	11.46	11.35
Annual Compound Return	9.76	9.78	9.80	9.85
Cumulative Wealth	52.43	52.69	53.10	54.29
N	214	120	68	39

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 8 Historic Performance of Rebalancing Portfolio with 2% threshold (1970-2012)

	Monthly	Quarterly	Semi-Annual	Annual
Mean Return	0.82	0.82	0.83	0.82
Volatility of Return	2.75	2.73	2.72	2.74
Minimum Return	-10.20	-10.78	-10.34	-11.49
Maximum Return	12.01	11.96	11.35	11.35
Annual Compound Return	9.80	9.84	9.88	9.85
Cumulative Wealth	53.20	53.98	54.91	54.13
N	83	56	45	34

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 9 Historic Performance of Rebalancing Portfolio with 4% threshold (1970-2012)

	Monthly	Quarterly	Semi-Annual	Annual
Mean of Return	0.83	0.82	0.83	0.83
Volatility of Return	2.75	2.74	2.73	2.75
Minimum Return	-10.78	-10.78	-10.34	-11.49
Maximum Return	12.01	11.96	11.35	11.35
Annual Compound Return	9.90	9.80	9.96	9.89
Cumulative Wealth	55.35	53.24	56.52	55.14
N	32	27	25	18

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 10 Historic Performance of Rebalancing Portfolio with 5% threshold (1970-2012)

	Monthly	Quarterly	Semi-Annual	Annual
Mean Return	0.83	0.82	0.83	0.82
Volatility of Return	2.74	2.74	2.74	2.75
Minimum Return	-10.78	-11.49	-11.49	-11.49
Maximum Return	12.01	11.61	11.61	11.76
Annual Compound Return	9.96	9.81	9.89	9.87
Cumulative Wealth	56.52	53.34	55.09	54.53
N	27	18	17	14

The mean return, volatility of return, maximum return, and minimum return are stated in percent per month. Cumulative Wealth is the ending value of one dollar invested in account at the beginning of the evaluation period.
N refers the number of times the portfolio was rebalanced.

Table 11 Cumulative wealth of portfolio with different thresholds and frequencies (1970-2012)

Threshold	Monthly	Quarterly	Semi-Annual	Annual
1%	52.43	52.68	53.1	54.29
2%	53.20	53.98	54.91	54.13
3%	53.46	54.11	54.76	54.45
4%	55.35	53.24	56.52	55.14
5%	56.52	53.34	55.09	54.53

The column category is the threshold of rebalancing. The cumulative wealth of a "no rebalancing" simulation is 44.38.

Table 12 Cumulative wealth comparison between rebalanced and no rebalanced portfolio (1975 - 2012)

	Initial	1975	1980	1985	1990	1995	2000	2005	2010	2012
Annual	1	1.22	2.56	4.96	10.61	16.25	34.76	39.65	52.99	54.45
No Rebalancing	1	1.21	2.44	4.61	10.4	15.53	34.07	34.48	39.12	44.37

Table 13 Portfolio evolution comparison between rebalanced and no rebalanced portfolio (2000-2005)

	Canadian Equities	US Equities	EAFE Equities	Global Equities	Fixed Income	Portfolio Value
Jan-00	8.69	5.21	5.21	3.48	12.17	34.76
Jan-02	8.55	5.13	5.13	3.42	11.97	34.19
Jan-03	7.75	4.65	4.65	3.1	10.85	31.01
Jan-04	9.31	5.59	5.59	3.73	13.04	37.26
Jan-05	10.24	5.56	6.09	3.85	13.92	39.66
Gain & Loss	1.55	0.34	0.87	0.38	1.75	4.89
Total Return	18%	7%	17%	11%	14%	14%
Jan-00	5.88	9.1	7.77	4.72	6.62	34.08
Jan-02	5.45	8.3	5.85	3.87	7.94	31.41
Jan-03	4.76	6.12	4.81	3	8.55	27.24
Jan-04	6.3	7.18	6.15	3.65	9.32	32.6
Jan-05	6.92	7.14	6.7	3.78	9.95	34.49
Gain & Loss	1.05	-1.95	-1.07	-0.94	3.33	0.41
Total Return	18%	-21%	-14%	-20%	50%	0%

Table 14 Simulations based on historic means with 2% threshold

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.82	0.82	0.82	0.82
Volatility of Return	2.78	2.78	2.78	2.78
Minimum Return	-7.23	-7.24	-7.24	-7.25
Maximum Return	9.68	9.68	9.69	9.70
Annual Compound Return	9.78	9.78	9.78	9.78
Cumulative Wealth	63.97	63.98	63.98	64.02
Minimum Cumulative Wealth	3.89	3.91	3.90	3.82
5% Quantile of Cumulative Wealth	18.83	18.78	18.74	18.87
Median of Cumulative Wealth	52.87	52.86	52.86	52.77
95% Quantile of Cumulative Wealth	146.32	146.00	146.88	146.93
Maximum Cumulative Wealth	569.93	574.82	588.24	603.09
Volatility of Cumulative Wealth	44.69	44.72	44.77	44.95
N	87.94	64.55	47.34	30.79

The returns in simulation have the same mean and variance covariance as the historic data.
Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 15 Simulations based on historic means with base case 3% threshold

	Monthly	Quarterly	Semi Annual	Annual	No Rebalance
Mean Return	0.82	0.82	0.82	0.82	0.82
Volatility of Return	2.78	2.78	2.78	2.78	2.84
Minimum Return	-7.24	-7.24	-7.25	-7.26	-7.65
Maximum Return	9.68	9.69	9.69	9.70	9.99
Annual Compound Return	9.78	9.78	9.78	9.78	9.77
Cumulative Wealth	63.98	63.98	64.01	64.03	66.09
Minimum Cumulative Wealth	3.85	3.88	3.97	3.85	6.83
5% Quantile of Cumulative Wealth	18.77	18.78	18.81	18.79	19.57
Median of Cumulative Wealth	52.83	52.74	52.85	52.66	50.38
95% Quantile of Cumulative Wealth	146.63	146.42	146.39	146.25	162.72
Maximum Cumulative Wealth	568.68	577.90	581.04	599.30	1452.22
Volatility of Cumulative Wealth	44.74	44.79	44.82	44.95	59.19
N	45.26	36.33	29.63	22.31	0.00

The returns in simulation have the same mean and variance covariance in history.
Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 16 Simulations based on historic means with 4% threshold

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.82	0.82	0.82	0.82
Volatility of Return	2.78	2.78	2.78	2.78
Minimum Return	-7.24	-7.25	-7.25	-7.26
Maximum Return	9.69	9.70	9.70	9.71
Annual Compound Return	9.78	9.78	9.78	9.78
Cumulative Wealth	64.02	64.03	64.04	64.07
Minimum Cumulative Wealth	3.80	3.85	3.81	3.89
5% Quantile of Cumulative Wealth	18.79	18.79	18.84	18.81
Median of Cumulative Wealth	52.71	52.67	52.71	52.71
95% Quantile of Cumulative Wealth	146.88	146.79	147.27	147.25
Maximum Cumulative Wealth	582.92	589.76	605.07	621.81
Volatility of Cumulative Wealth	44.84	44.87	44.90	45.04
N	27.38	23.12	19.67	15.98

The returns in simulation have the same mean and variance covariance in history.
Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 17 Simulation based on hypothetical means with 2% threshold
(Annual Compound Returns: Canadian Equities 9.5%, US Equity 10%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 7%)

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.75	0.75	0.75	0.75
Volatility of Return	2.78	2.78	2.78	2.78
Minimum Return	-7.30	-7.31	-7.31	-7.33
Maximum Return	9.63	9.64	9.65	9.67
Annual Compound Return	8.86	8.86	8.86	8.86
Cumulative Wealth	44.62	44.65	44.70	44.81
Minimum Cumulative Wealth	2.52	2.52	2.51	2.53
5% Quantile of Cumulative Wealth	13.06	13.03	13.05	13.01
Median of Cumulative Wealth	36.89	36.90	36.89	36.96
95% Quantile of Cumulative Wealth	103.20	103.18	103.33	103.41
Maximum Cumulative Wealth	379.59	383.36	384.89	391.43
Volatility of Cumulative Wealth	30.59	30.66	30.76	30.99
N	88.18	64.69	47.50	30.91

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 18 Simulation based on hypothetical means with base case 3% threshold (Annual Compound Returns: Canadian Equities 9.5%, US Equity 10%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 7%)

	Monthly	Quarterly	Semi Annual	Annual	No Rebalance
Mean Return	0.75	0.75	0.75	0.75	0.77
Volatility of Return	2.78	2.78	2.78	2.79	3.10
Minimum Return	-7.31	-7.32	-7.32	-7.34	-8.42
Maximum Return	9.64	9.65	9.66	9.68	10.85
Annual Compound Return	8.86	8.86	8.86	8.86	9.07
Cumulative Wealth	44.68	44.70	44.75	44.85	53.49
Minimum Cumulative Wealth	2.56	2.55	2.54	2.51	2.54
5% Quantile of Cumulative Wealth	13.09	13.03	13.06	12.94	12.31
Median of Cumulative Wealth	36.93	36.91	36.87	37.02	38.44
95% Quantile of Cumulative Wealth	103.36	103.36	103.71	104.00	144.76
Maximum Cumulative Wealth	383.24	390.49	390.23	390.04	963.63
Volatility of Cumulative Wealth	30.69	30.77	30.88	31.07	50.80
N	45.56	36.55	29.84	22.48	0.00

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 19 Simulation based on hypothetical means with 4% threshold (Annual Compound Returns: Canadian Equities 9.5%, US Equity 10%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 7%)

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.75	0.75	0.75	0.75
Volatility of Return	2.78	2.78	2.78	2.79
Minimum Return	-7.32	-7.33	-7.34	-7.35
Maximum Return	9.65	9.66	9.67	9.70
Annual Compound Return	8.86	8.86	8.86	8.87
Cumulative Wealth	44.75	44.79	44.85	44.95
Minimum Cumulative Wealth	2.53	2.50	2.55	2.48
5% Quantile of Cumulative Wealth	13.02	13.00	13.01	12.94
Median of Cumulative Wealth	36.88	36.96	36.87	36.88
95% Quantile of Cumulative Wealth	103.38	103.91	104.27	104.24
Maximum Cumulative Wealth	382.00	402.79	395.40	395.17
Volatility of Cumulative Wealth	30.85	30.96	31.09	31.27
N	27.61	23.27	19.84	16.15

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 20 Simulation based on hypothetical means with 2% threshold (Annual Compound Returns: Canadian Equities 9.5%, US Equity 20%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 5%)

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.80	0.81	0.81	0.81
Volatility of Return	2.78	2.78	2.79	2.80
Minimum Return	-7.26	-7.27	-7.28	-7.31
Maximum Return	9.70	9.71	9.73	9.78
Annual Compound Return	9.61	9.63	9.65	9.70
Cumulative Wealth	59.93	60.41	61.05	62.33
Minimum Cumulative Wealth	5.11	5.02	5.10	5.04
5% Quantile of Cumulative Wealth	17.49	17.63	17.73	18.01
Median of Cumulative Wealth	48.89	49.23	49.74	50.51
95% Quantile of Cumulative Wealth	139.55	140.37	141.88	145.46
Maximum Cumulative Wealth	742.89	737.20	770.71	799.55
Volatility of Cumulative Wealth	42.48	42.89	43.53	44.80
N	90.47	66.83	49.22	32.09

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 21 Simulation based on hypothetical means with base case 3% threshold (Annual Compound Returns: Canadian Equities 9.5%, US Equity 20%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 5%)

	Monthly	Quarterly	Semi Annual	Annual	No Rebalance
Mean Return	0.81	0.81	0.81	0.82	1.23
Volatility of Return	2.78	2.79	2.79	2.80	3.63
Minimum Return	-7.27	-7.28	-7.29	-7.32	-9.48
Maximum Return	9.72	9.73	9.75	9.79	13.49
Annual Compound Return	9.65	9.67	9.70	9.74	15.00
Cumulative Wealth	61.01	61.60	62.29	63.51	581.34
Minimum Cumulative Wealth	5.08	5.09	5.11	5.10	10.36
5% Quantile of Cumulative Wealth	17.81	17.99	18.11	18.31	82.19
Median of Cumulative Wealth	49.64	50.18	50.65	51.53	377.21
95% Quantile of Cumulative Wealth	142.03	143.64	145.19	148.35	1770.84
Maximum Cumulative Wealth	747.26	759.94	779.21	804.36	11805.08
Volatility of Cumulative Wealth	43.34	43.83	44.49	45.65	691.35
N	47.95	38.95	32.03	24.37	0.00

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.

Table 22 Simulation based on hypothetical means with 4% threshold
(Annual Compound Returns: Canadian Equities 9.5%, US Equity 20%, EAFE Equities 8%, Global Equities 9%, Canadian Fixed Income 5%)

	Monthly	Quarterly	Semi Annual	Annual
Mean Return	0.81	0.81	0.82	0.82
Volatility of Return	2.79	2.79	2.80	2.81
Minimum Return	-7.28	-7.29	-7.31	-7.33
Maximum Return	9.74	9.76	9.78	9.82
Annual Compound Return	9.71	9.73	9.76	9.81
Cumulative Wealth	62.38	63.01	63.74	65.04
Minimum Cumulative Wealth	5.09	5.03	5.20	5.23
5% Quantile of Cumulative Wealth	18.18	18.41	18.49	18.86
Median of Cumulative Wealth	50.76	51.23	51.85	52.73
95% Quantile of Cumulative Wealth	145.30	146.74	148.94	152.74
Maximum Cumulative Wealth	766.01	767.47	792.52	824.82
Volatility of Cumulative Wealth	44.42	44.92	45.59	46.76
N	30.20	25.84	22.25	18.37

Mean return, volatility of return, minimum return, maximum return, annual compound return, cumulative wealth and N refer to the average of those in 10,000 times simulation.